Real Business Cycle Theory

Motivation, Model & Discussion

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SS Macroeconomic Theory

Lecture Outline

Introduction to RBC Models

RBC: Freshwater Macroeconomics

- Real shocks with technology important in short run.
- **Temporary** but **somewhat persistent** shocks to technology.
- Cobb-Douglas production function: $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$.
- Interpreting A.
- AR(1) process $\Longrightarrow$ persistent.
- Example.

Introduction to RBC Models

Example

- $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} = C_t + I_t$
- $\Delta A_t \longrightarrow \Delta Y_t$ (potential ouptut).
- RBC always $Demand = Y_t$ and $r_t = r_n$ versus New Keynesian model.
- Flexible prices $\Longrightarrow$ market clearing $\Longrightarrow D = S$.
- E.g. once-off $\uparrow A \longrightarrow \uparrow Y_t \stackrel{PIH(temp)}{\longrightarrow} \uparrow S \longrightarrow \uparrow K \& I$.
- C rises a little, I rises a lot.
- So, moderately procyclical C; highly procyclical and volatile I.
- $\uparrow K_t \longrightarrow \uparrow K_{t+1} \longrightarrow \uparrow C_{t+1}, \uparrow I_{t+1}$, etc.
- $\therefore$ Persistent economic fluctuations.

Introduction to RBC Models

Example continued

- $w = MPL = \frac{\partial Y}{\partial L}, \ \ \frac{\partial}{\partial A}\frac{\partial Y}{\partial L} > 0.$
- Therefore, temporary
  $\uparrow A \longrightarrow \uparrow MPL \longrightarrow \uparrow L^D \longrightarrow \uparrow \frac{W}{P} \xrightarrow{IEL^S} \uparrow L^S.$
- So the production function $Y \uparrow$, getting an extra 'kick' from $L \uparrow$.

Introduction to RBC Model

Persistence, Substitution Effects & Income Effects

- Changes in A are temporary but modestly persistent.
- $g_{A_t} = 2\%$ shocked so $g_{A_{t+1}} = 4\%$, $g_{A_{t+2}} = 3\%$, $g_{A_{t+3}} = 2\%$.
- Completely temporary: save almost all extra income (PIH).
- To generate fairly procyclical consumption, need persistence (feel richer, but not *that* much richer). Just a modelling device? No, actually plausible.
- Assume $SE > IE$ so $\uparrow A_t \longrightarrow \uparrow L_t \& \uparrow S_t$. Why? Temporary nature of shocks.

Introduction to RBC Model

Propogation Mechanisms

Propogation Mechanism: model's internal way of amplifying the shock.

New Keynesian model? **Sticky prices**. Financial accelerator (see footnote 2 in chapter 6 notes).

RBC model:

1. **Intertemporal substitution of labour supply**.
   $\uparrow A \overset{MPL=MB>MC=\frac{W}{P}}{\longrightarrow} \uparrow L^d \longrightarrow \uparrow \frac{W}{P} \longrightarrow \uparrow L^S$. Labour supply also rises since $\uparrow A \longrightarrow MPK \longrightarrow \uparrow r_n \longrightarrow L^S \uparrow$ since can earn greater return by purchasing capital and renting it out the *next* period.
2. **Capital accumulation**:
   $\uparrow I_t \longrightarrow \uparrow K_{t+1} \longrightarrow \uparrow Y_{t+1} \longrightarrow \uparrow I_{t+1}$. So capital accumulation 'propogates' the shock. Also $\uparrow K_{t+1} \longrightarrow \uparrow MPK_{t+1} \longrightarrow \uparrow \frac{W_{t+1}}{P_{t+1}} \longrightarrow \uparrow L^S_{t+1}$.

Lecture Outline

RBC Model

Formulation

$$E_0 \sum_{t=0}^{t=\infty} \beta^t \left(\log C_t - \frac{l_t^{1+\sigma}}{1+\sigma} \right) \quad \beta = \frac{1}{1+\rho}$$

$$w_t l_t + r_t k_t = C_t + i_t$$

$$k_{t+1} = (1-\delta) k_t + i_t$$

$$l_t + h_t = 1$$

RBC Model

Optimality

$$L = \sum_{t=0}^{t=\infty} \beta^t \left(\log C_t - \frac{l_t^{1+\sigma}}{1+\sigma} \right) + \sum_{t=0}^{t=\infty} \lambda_t (w_t l_t + (1+r_t) k_t - C_t - k_{t+1})$$

$$[C_t] : \beta^t \frac{1}{C_t} = \lambda_t$$

$$[l_t] : \beta^t l_t^{\sigma} = \lambda_t w_t$$

$$[k_{t+1}] : -\lambda_t + \lambda_{t+1}(1 + r_{t+1}) = 0$$

Transversality condition: $\lim_{t \longrightarrow \infty} E_0 \beta^t u'(C_t) k_t = 0$.

RBC Model

Solution under Certainty

Set $\delta = 0$ so r_t is real return net of depreciation. $[C_t]$, $[C_{t+1}]$ and $[k_{t+1}]$ yields Euler equation

$$\frac{1}{C_t} = E_t \beta (1 + r_{t+1}) \frac{1}{C_{t+1}}$$

Combining $[C_t]$ & $[l_t]$:

$$w_t u'(C_t) = v'(l_t) \iff \frac{w_t}{C_t} = l_t^{\sigma}$$

RBC Model

Intertemporal Substitution of Labour

$$\text{Divide } \frac{w_{t+1}}{C_{t+1}} = l_{t+1}^{\sigma} \text{ by } \frac{w_t}{C_t} = l_t^{\sigma} \text{ to get } \frac{l_{t+1}^{\sigma}}{l_t^{\sigma}} = \frac{w_{t+1}}{w_t}\frac{C_t}{C_{t+1}} \tag{1}$$

Ignore uncertainty so Euler equation is

$$\frac{1}{C_t} = \beta(1 + r_{t+1})\frac{1}{C_{t+1}} \tag{2}$$

Set $\beta(1 + r_{t+1}) = 1$ and use (2) in (1) to get

$$\frac{l_{t+1}^{\sigma}}{l_t^{\sigma}} = \frac{w_{t+1}}{w_t}$$

$$\therefore \frac{l_{t+1}}{l_t} = \left(\frac{w_{t+1}}{w_t}\right)^{\frac{1}{\sigma}}$$

Interpretation – 'seduced' to deviate – DMU / Frisch elasticity $\frac{1}{\sigma}$.

RBC Model

(i) Intertemporal Substition of Labour & Interest Rates (ii) Labour Supply Levels

(i) Also interest rates ($\beta(1+r_{t+1}) \neq 1$):

$$\frac{l_{t+1}}{l_t} = \left(\frac{w_{t+1}}{w_t} \frac{1}{\beta(1+r_{t+1})} \right)^{\frac{1}{\sigma}}$$

(ii) Labour Supply Levels:

$$\frac{w_t}{C_t} = l_t^{\sigma} \Longrightarrow l_t = \left(\frac{w_t}{C_t} \right)^{\frac{1}{\sigma}}$$

Graph.

$$l_{t+1} = \left(\frac{w_{t+1}}{C_{t+1}} \right)^{\frac{1}{\sigma}}$$

Next: The Firm.

RBC Model

The Firm

- New Keynesian firms: imperfectly competitive $P > MC$.
- RBC firms: perfectly competitive, so price takers, choose L, K.
- Normalise price to one so w, r are real.
- CAPITAL LETTERS refer to DEMAND (from FIRM): K, L, while lower case letters refer to supply (from household): k, l.

RBC Model

The Firm

$$\text{Profit } \Pi = \underbrace{AK^{\alpha}L^{1-\alpha}}_{\text{revenue}} - \underbrace{wL - rK}_{\text{costs}}$$

- Labour demand: equate $\frac{\partial \Pi}{\partial L} = 0$, will get $w = MPL$ and solve for L

$$L^d = \left(\frac{(1-\alpha)AK^{\alpha}}{w} \right)^{\frac{1}{\alpha}}$$

- Capital demand: equate $\frac{\partial \Pi}{\partial K} = 0$, will get $r = MPK$ and solve for K

$$K = \left(\frac{\alpha AL^{1-\alpha}}{r} \right)^{\frac{1}{1-\alpha}}$$

RBC Model

General Equilibrium

- Labour & capital demand (supply) from firm (household), so combine both sides to obtain market clearing / general equilibrium: $l_t = L_t$ and $k_t = K_t$.
- Take price and wage as given (not internalise).
- Interaction of labour supply and demand yields w^*, which equals MPL.
- Interaction of downward sloping K and vertical k yields r^*, which equals MPK.
- $w^*l + r^*k = Y = Ak^\alpha l^{1-\alpha}$: payments to factors of production exhaust output.

RBC Model

Long-Run Equilibrium

- Assume no steady state growth in long run, so labour supply is constant at

$$l_t = \left(\frac{w_t}{C_t}\right)^{\frac{1}{\sigma}}$$

Sustained growth in A would lead to equal growth of C and w so l would still be constant. Labour hours approximately constant in recent decades, so model is consistent with long-run trends.

RBC Model

Long-Run Equilibrium

- To find equilibrium capital ratio, Euler equation:

$$\frac{1}{C_t} = \beta(1 + r_{t+1})\frac{1}{C_{t+1}}$$

No growth in steady state $C_t = C_{t+1} \Longrightarrow \beta(1 + r_{t+1}) = 1$ so rental rate is constant $r^* = \alpha A k^{\alpha-1} l^{1-\alpha}$. So

$$\beta(1 + r_{t+1}) = 1 \Longrightarrow \beta(1 + \alpha A k^{\alpha-1} l^{1-\alpha}) = 1 \Longrightarrow \frac{k}{l} = \left(\frac{\alpha A}{\frac{1}{\beta} - 1}\right)^{\frac{1}{1-\alpha}}$$

Lecture Outline

Discussion

Are fluctuations desirable?

- Fluctuations represent optimal responses to economic conditions.
- Flucutations are Pareto Optimal (follows from perfect competition assumption and First Welfare Theorem).

Discussion

Review of Dynamics in RBC

What happens when there is a technology shock? Mechanisms...

- Temporary reduction in level of regulation.
- A rises (temporary and somewhat persistent – rational expectations).
- $A \uparrow \longrightarrow MPL \uparrow \longrightarrow \uparrow L^d \uparrow \frac{W}{P} \longrightarrow l$ (SE>IE).
- $\uparrow l \& \uparrow A \longrightarrow \uparrow MPK \longrightarrow \uparrow$ demand for capital $\longrightarrow \uparrow r$.
- Also, expected r next period rises (changes are persistent) so $I \uparrow$ this period.
- Household income rises but temporary nature of shock means most extra income is saved and used for purchasing more capital.
- $\uparrow E_t(r_{t+1}) \longrightarrow \uparrow S_t$.

Discussion

Review of Dynamics in RBC

- Persistent change $\Longrightarrow C \uparrow$ a little. $Y = C + I$ so since $Y \uparrow$ a lot ($\uparrow (A, L)$) while C only rises a little, I (or savings) rises a lot. Households 'smooth consumption intertemporally by building up their capital stock.'
- $\uparrow I_t \longrightarrow K_{t+1} \uparrow$. A_{t+1} falls a bit but still above trend. K_{t+1} higher due to last period's investment boom. Both reasons imply wage still above trend, so labour supply still above trend.
- A reverts to trend and shock 'dies out'. Addition to K will also depreciate and capital reverts back to its 'normal' level. All mechanims die out and economy reverts to trend.

Discussion

Empirical Evidence – Important Section

Use time series data Y, K, A and labour and capital shares of income to get Solow residual $A = \frac{Y}{K^{\alpha}L^{1-\alpha}}$. Labour productivity will be

$$\frac{Y}{L} = \frac{AK^{\alpha}L^{1-\alpha}}{L} = \frac{AK^{\alpha}}{L^{\alpha}}$$

RBC: labour productivity will be procyclical (since A rises in booms and falls in recessions). New Keynesians: labour productivity is countercyclical in standard model ($L \uparrow \longrightarrow \frac{Y}{L} \downarrow$); acyclical in our model since we assumed CRS to labour.

Discussion

Empirical Evidence – Important Section

RBC: Solow residual is exogenous and induces movements in Y. NK: Solow residual is endogenous to the cycle (e.g. increasing returns). Reverse causation. Restaurants and labour hoarding (lunchtime) – is a technology shock causing the increase in output? Hardly! Demand fluctuations. Utilisation rates.

$$Y_t = A(u_t K_t)^\alpha (u_t L_t)^{1-\alpha} = A u_t K_t^\alpha L_t^{1-\alpha}$$

u: utilisation rate/effort endogenous to cycle; high u is like high A. NK: changes in A are really changes in u, so $\frac{Y}{L}$ can be procyclical without exogenous changes in A. RBC response: Y endogenously causes M – delivers a 'knock out' to Keynesian view that M causes fluctuations in Y.

Discussion

Stabilisation Policy – Important Section

- Stabilisation policy: reduce volatility of business cycles, e.g. raising interest rates can help reduce investment in booms and encourage investment in recession.
- RBC: stabilisation policy is counterproductive (recall first slide of this section) – reduces welfare. RBC: business cycles are optimal responses/efficient.
- Transferring resources from productive periods to unproductive periods. Why try to induce less work when you are full of energy and *vice-versa*?

Discussion

Stabilisation Policy – Important Section

- RBC: money neutrality, but fiscal policy effective: $\uparrow G \longrightarrow \uparrow L^s$ and $\downarrow C$ (income effect: feel poorer since Y fixed by potential but pay more taxes in future via Ricardian equivalence so lifetime wealth $\downarrow$).
- NK: $\uparrow G \longrightarrow \uparrow$ demand and so $Y \uparrow$.
- So, different mechanism in RBC than channels in NK model. RBC always stresses incentives to supply labour and investment.

Lecture Outline

Doing this in the real world

MATLAB & Dynare

Pseudo Algorithm

1. **Formulate** your model and write down the optimality conditions.
2. See if you can solve for *steady state* – trick ($l = \frac{1}{3}$).
3. Code it up: calibrate parameters and **solve** model.
4. **Estimate** model and use for policy purposes, etc.

- Lots of hidden steps and tricks, solution methods, estimation methods.
- In practice, Dynare, an engine for MATLAB makes this (sometimes) relatively painless.

MATLAB & Dynare

- MATLAB: Trinity has site license – can use in computer labs around college.
- GNU Octave is a free alternative to MATLAB – see `www.tcd.ie/Economics/staff/frainj/main/freeSoftware/freeSoftware.html`
- Dynare is a free plug-in for MATLAB and GNU Octave that allows us to automate a lot of the process of working with Dynamic Stochastic General Equilibrium (DSGE) models: `www.dynare.org`
- Dynare Wiki has easily accessible instructions on how to get started with Dynare (installation, basics of DSGEs for economists, etc.)
- Lots of code on World Wide Web to replicate papers, etc. using Dynare.
- A great tool, though beware: (i) Dynare monkeys; (ii) for non-linear work especially past 2nd/3rd orders, Dynare is, at best, 'limited' (need to go further: Mathematica or Gauss or MATLAB's Symbolic Toolbox. . . maybe even FORTRAN – college has all of these).

MATLAB & Dynare

Example 1 – rbc1b

$$E_0 \sum_{t=0}^{t=\infty} \beta^t \left(\frac{C_t^{1-\theta}}{1-\theta} - \gamma l_t \right)$$

$$C_t^{-\theta} \overset{EE}{=} \beta C_{t+1}^{-\theta} (\underbrace{\alpha e^{z_{t+1}} k_{t+1}^{\alpha-1} l_{t+1}^{1-\alpha}}_{r_{t+1}} + 1 - \delta)$$

$$\gamma = C_t^{-\theta}(1-\alpha) e^{z_t} k_t^{\alpha} l_t^{-\alpha} = \underbrace{C_t^{-\theta}}_{u'(C_t)} \underbrace{(1-\alpha)\frac{y_t}{l_t}}_{w_t}$$

$$C_t + k_{t+1} = \underbrace{e^{z_t} k_t^{\alpha} l_t^{1-\alpha}}_{y_t} + (1-\delta) k_t$$

$$y_t = e^{z_t} k_t^{\alpha} l_t^{1-\alpha}$$

$$z_t = \rho z_{t-1} + \epsilon_t$$

Example 1 – rbc1b

Steady State

- Assuming $z = 0$ and $l = \frac{1}{3}$, in steady state assume C, k, l, y are constant or deviations are zero.
- Can reduce equations on previous period to solve for C, k, l, y in terms of parameters α, β, δ: solve for k first from Euler equation and then get C from resource constraint and y from production function.
- k and l are states: can figure out the evolution of all variables by knowing these two variables.

Example 1 – rbc1b

Steady State

$$k = \frac{1}{3}\left(\frac{1}{\alpha}\left(\frac{1}{\beta} - 1 + \delta\right)\right)^{-\frac{1}{1-\alpha}}$$

$$C = k^{\alpha}\left(\frac{1}{3}\right)^{1-\alpha} - \delta k$$

$$y = k^{\alpha}\left(\frac{1}{3}\right)^{1-\alpha}$$

MATLAB & Dynare

Example 2 – rbc2

$$\max E \sum_{t=0}^{\infty} \beta^t \{\log c_t + \psi \log (1 - l_t)\}$$

- We will look at the relative volatility of consumption and investment with respect to output. It turns out that: $\sigma_C < \sigma_Y < \sigma_I$ for most industrialised countries ($0.43 < 1.35 < 4.66$); see dynare files (check your email).
- Interestingly, different story for emerging markets: $\sigma_Y < \sigma_C < \sigma_I$, i.e. $\sigma_Y < \sigma_C$. Fewer insurance mechanisms for smoothing consumption, e.g. access to capital markets, financial products, expensive. . .
- Next: Impulse Response Functions (IRFs) trace the evolution of variables ('functions') as a 'response' to a shock ('impulse').

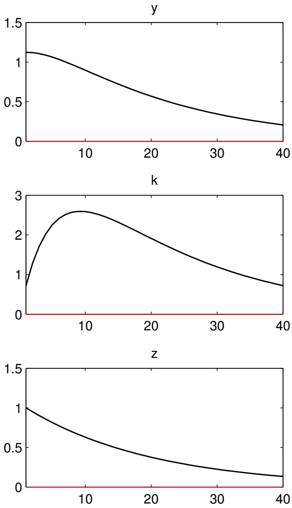

Figure: Example one IRFs (response to productivity shock).

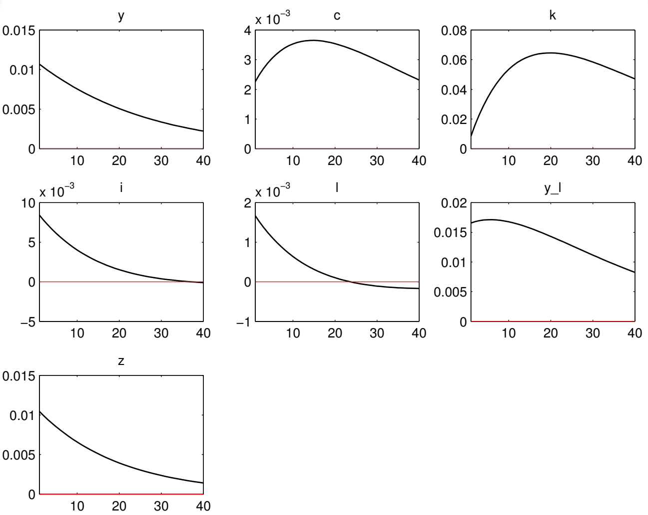

Figure: Example two IRFs (response to productivity shock).