Problem Set 6: Real Business Cycle Model

Real Business Cycle Theory

Exercise 1. Suppose a government decides to raise tax rates permanently. However, it *phases* in the new policy over a number of years. What are the economic effects of this for labour supply?

Solution 1 (Tax Smoothing)**.** The theory of *tax smoothing* would suggest raising tax rates permanently *immediately* rather than phasing them in. If the tax rates are gradually hiked, tax-payers know that taxes will be higher each period and so there will be a substitutions effect towards working in each period rather than in the future but less and less each time. If the taxes are permanent, the income effect dominates and so since peope feel poorer, they work more permanently.

Exercise 2. Empirically, *quits* from jobs fall in recessions. Is this observation consistent with RBC theory?

Solution 2 (Evidence regarding RBC theory)**.** According to RBC-ers, recessions reflect negative fluctuations in technology (the Solow residual), i.e. growth in technology is below its trend. When productivity goes down, it is partially because those who may have quit before and are unproductive do not quit. Layoffs may rise if there is no labour hoarding, or fall/stay constant if there is labour hoarding. You might not want to work more when you are unproductive according to RBC-ers, so the government should not step in to stabilise the economy.

Exercise 3. Write down the labour/leisure optimality condition in the RBC model. Over time, wage and consumption grow at approximately the same rate. What does the RBC model predict for the long-run trend in labour hours?

Solution 3 (Long-run labour trend in RBC)**.** The labour/leisure optimality condition is $w_t u'(C_t) = v'(l_t)$ for the RBC model or more explicitly $\frac{w_t}{C_t} = l_t^\sigma$. We can also get $\frac{l_{t+1}}{l_t} = \left(\frac{w_{t+1}}{w_t}\right)^{\frac{1}{\sigma}}$. From $\frac{w_t}{C_t} = l_t^\sigma$, if wages and consumption grow at approximately the same rate, then the ratio $\frac{w_t}{C_t}$ will remain constant so l_t^σ will remain constant, i.e. the RBC model predicts that labour hours remain constant as a trend line in long-run. Since we observe in the data that labour hours are roughly constant over time, the RBC does a good job here of matching the data.

Exercise 4. Does RBC theory take the view that recessions are desirable?

Solution 4 (Desirability of recessions)**.** Flucations are optimal responses to changes in technology. Fluctuations are Pareto Optimal, which follows from perfect competition assumption and first welfare theorem. RBC theory interprets recessions as arising from deviations of TFP growth below its trend. Productivity is fundamentally higher in booms and lower in recessions. Stabilisation policy is counterproductive since it is transferring resources/production from productive periods to least productive periods so reduces welfare. When we are in great form and full of energy, we should work harder; when we are down and lazy, we should stay in bed. So, the business cycle represents optimal responses by economic agents to changes in their economic environment, i.e. booms and recessions are efficient.

Exercise 5. Suppose the Irish government raises marginal tax rates on wages *temporarily* in the budget. Explain clearly the implications of this policy would within the context of the RBC model.

Solution 5 (Topic)**.** Assume a one period shock to tax rates on wages. If the government were to raise marginal tax rates on wages temporarily, the substitution effect would tend to dominate the income effect and both the intertemporal substitution and capital accumulation channels would propogate the shock

downwards negatively; for the capital accumulation channel, the fact that there would be less labour supply means that capital would be less productive (each machine becomes less useful) reducing the demand for capital. Household income falls but temporary nature of the shock means that consumers won't feel that much poorer so will only reduce consumption by a small amount. Since they anticipate that the tax will be temporary, they smoothly adjust their consumption over time so the fall in consumption will be smaller the more temporary the shock is. Labour supply falls initially due to the intertemporal substitution effect but will rise back towards its initial level from the second period onwards due to the substitution effect (when the tax falls back towards its initial value) and the negative income effect. Supply of capital is fixed but as the firms now use less capital and less labour and technology is constant, output will fall temporarily. As people work more from the second period onwards, capital productivity returns to normal and capital demand returns to normal so with constant technology, output returns to normal. To the extent that wages are perfectly flexible, when people initially want to reduce their labour supply as labour taxes rise, their is upward pressure on real wages since labour supply has diminishing marginal productivity. This may induce them not to reduce labour supply too much, but they will reduce labour supply a little since if they did not reduce labour supply then firms would have to pay a higher real wage than initially to compensate workers for the wage tax hike even though their was no increase in productivity (no reduction in labour supply). When capital falls initially, MPK rises due to diminishing marginal productivity of capital. This pushes upwards pressure on rental rates and encourages investment once again.

Exercise 6. According to RBC theory, if the government was attempting to stabilize labour input over the business cycle, how should it levy taxes on wage income?

Solution 6 (Topic). The government should not try to stabilise labour over the business cycle – fluctuations are already Pareto Optimal responses to technology shocks. According to tax smoothing, the government should not phase taxes in or out but change them permanently immediately; otherwise, people would substitute labour supply over time with rational expectations that taxes were going to be higher/lower in the future.

Exercise 7. Suppose that, because of budget deficits, the government systematically raises labour taxes in a recession, but lowers them in a boom. What are the implications for labour supply?

Solution 7 (Topic). In a recession, with technology lower, people should work less according to RBC. Raising labour taxes will amplify this effect and likewise (work more) in a boom. Labour supply will become even more procyclical. Labour taxes like these will strengthen the intertemporal substitution of labour supply.

Exercise 8. Explain why, the more transitory the shock, the greater the increase in the investment/gdp ratio, when A rises.

Solution 8 (Transitory Shocks). The more transitory the shock, the more the substitution effect dominates the income effect and so the more a rise in real wages (since the marginal benefit MPL to working rises when A rises) encourages labour supply. Household income rises and because this change is temporary, most of the increase in income is saved and used for purchasing more capital, i.e. an increase in I. This rise in the future expected interest rate (since more capital [takes one period to gestate] boosts MPK] induces the rise in savings. Yet, the more temporary the shock, the less consumption rises. Consider the national acounts equation $Y = C + I$. Since Y has increased a lot but the size of the rise in C is smaller the more transitory the shock, the bigger the rise in I the more transitory the shock and so the greater the increase in the investment/gdp ratio. E.g. start off with $I = 3$, $Y = 9$ so $C = 6$ and $\frac{I}{Y} = \frac{1}{3}$. Then a transitory shock may lead to $I = 6$ and $C = 7$ so $Y = 13$ and $\frac{I}{Y} = \frac{7}{13}$. A permanent shock would lead to a bigger rise in C and less of a rise in savings and so less of a rise in I.

Exercise 9. Suppose in a two period RBC model, utility takes the form

$$\log c_1 + \log c_2 - \gamma l_1 - \gamma l_2$$

while the lifetime budget constraint is $c_1 + c_2 = A_1 l_1 + A_2 l_2$. If $A_1 > A_2$, determine l_2. Use intuition only, and explain answer. Would your answer be the same if utility took the form $\log c_1 + \log c_2 - \gamma l_1^3 - \gamma l_2^3$. Explain.

Solution 9 (Topic). Note that the form of utility for labour is not that convex ($v''(l) = 0$ for $l \in \{l_1, l_2\}$) so intertemporal elasticity of substitution is quite high. Also, we care about the future the same as the present since discount factors are essentially one (also seen from lifetime budget constraint). [Note $\beta = \frac{1}{1+\rho}$ so rate of time preference is one]. As the first period technology is greater than the second, workers are more productive today so they should work more today. If the utility took the second form, the utility of labour would be more convex so intertemporal elasticity of substitution would be less high and so we while workers should work more today and less tomorrow, they would work less today and more tomorrow relative to the first case.

Exercise 10. Suppose the utility function in the RBC model takes the form

$$u(C) = \log C$$

Assume agents may work 12 hours per day. What would labour supply be in equilibrium? Explain how the dynamics of the model would change if there was a temporary, but persistent TFP shock.

Solution 10 (Role of Labour in Utility Function). Since labour does not enter the utility function, it will only be part of the budget constraint. Labour supply will be 12 hours per day in equilibrium since this maximises the income of the agents and comes at no utility cost since now leisure is not part of the utility function. The intertemporal substitution of labour is no longer a mechanism in the model so temporary increases in TFP would merely increase the wage and the marginal product of labour so labour demand would rise along with the real wage but labour supply could not increase (no intertemporal substitution of labour) since agents are already working at there maximum (12 hours per day), so Y would not get another kick from the increase in L. Therefore, the dynamics of the model would be muted.

Exercise 11. Explain what would happen to i) output ii) consumption and iii) labour supply if there was a temporary increase in A for one period, and the production function was $Y = AL$? Assume output is non-storable.

Solution 11 (No capital and non-storable output). As the marginal product of labour (MPL) is increasing in A, MPL (equals wage) raises labour demand and in turn the real wage. Through intertemporal substitution of labour, labour supply rises (particularly so because the temporary increase in A is one period so substitution effects dominate income effects). Even though there is no capital accumulation, the intertemporal substition of labour mechanism gives the production function Y another kick from this increase in L. Therefore, output increases. As output is non-storable, consumption will rise; note that if output was storable, then by the Permanent Income Hypothesis, since the rise in A (and hence Y) is temporary (one period), agents will save the extra income so consumption would not change (barely rises) to smooth consumption intertemporally. Labour only rises for one period and then returns to the level it was at prior to the increase in A. Likewise, output only rises for one period before returning to its previous level. The dynamics are the same for consumption.

Exercise 12. If shocks to A were not persistent, would investment still rise if A rose in a given period?

Solution 12 (Persistence of TFP Shocks & Investment)**.** If investment takes a period to transform into new capital, which could be used in production and combined with A, then there would be no point in investing since the rise in A would only improve the productivity of *existing* capital and by the time the new capital from investing more arrived, productivity would be at previous levels and if the previous level of capital was optimal, then there would be too much capital supplied relative to demand $K^d = \left(\frac{\alpha A L^{1-\alpha}}{r} \right)^{\frac{1}{1-\alpha}}$ and the marginal product of capital would be less than the rental rate the firm faces r^*.

Exercise 13. What would happen if consumers expected a once-off positive TFP shock *next period*? What implications would there be for the economy *this* period?

Solution 13 (Topic)**.** If consumers expect a once-off positive TFP shock next period, a rise in investment will occur today to create extra capital to be used with the more productive technology for that one period in the next period (MPK increases and hence natural rate of interest) so output will rise in that next period due to the capital accumulation mechanism. Likewise, the marginal product of labour will rise next period because of higher levels of capital and A so labour demand next period will rise along with real wages. This and the temporary nature of the shock will mean that substitution effects dominate income effects so that agents will intertemporally substitute labour by working more next period relative to this one though labour supply will return to prior levels thereafter along with capital accumulation. Of course, since the natural rate for capital will increase next period, consumers will also work more now since they can now earn a greater return by purchasing capital and renting it out in the next period. Output will get an extra kick because of the extra kick next period due to the rise in L. As long as output is storable, consumers will save most of the extra income next period due to the Permanent Income Hypothesis since lifetime wealth, which determines consumption in each periods, only increases a little bit from a temporary shock and ultimately lifetime wealth determines consumption levels in each period. So, labour supply increases in both periods but then returns to previous levels, investment increases in the period prior to the TFP shock so there is more capital for that period and that period only (investment returns to its previous level in the period of the TFP shock and thereafter). Output rises a little bit due to the extra labour supply in the first period and by even more in the second period (the one with the TFP shock) due to the extra labour supply and the extra capital and the boost in TFP but then returns to its previous level thereafter. Consumers save most of the extra income so consumption does not rise by much. Capital increases in the period with the TFP shock but is only higher in that period; capital returns to the previous level thereafter.

Exercise 14. If there was temporary but persistent increase in A, what would happen to the rental rate on capital over time.

Solution 14 (Evolution of Rental Rate of Capital)**.** A temporary persistent increase in A boosts the marginal productivity of capital and therefore the rental rate of capital rises. As A gradually returns to its previous level, the marginal productivity of capital reverts to its previous level and so too the rental rate of capital diminishes back to its original level.

Exercise 15. What happens if there was a *once-off* 1% increase in the capital stock in a given period?

Solution 15 (Rise in Capital Stock)**.** An increase in the capital stock in a given period would boost output through the production function but with diminishing returns to capital, the marginal productivity of capital would decline inducing a fall in the rental rate on capital. Since labour demand is given by $L^d = \left(\frac{(1-\alpha)AK^\alpha}{w} \right)^{\frac{1}{\alpha}}$ and K increases, labour supply must rise as upwards pressure on real wages would occur, further boosting output through through the production function; a rise in captial also boosts the marginal product of labour and thereby raises labour demand and wages in equilibrium. The rise in labour puts upward pressure on the rental rate of capital but the rise in capital would put a downward pressure on the rental rate of capital (reduction in MPK). Output would rise due to K and L with A constant.

The temporary nature would spur consumption to rise (SE dominating IE) though this would be small (smoothing / permanent income hypothesis). Since the rise in capital was once-off, labour demand would revert to normal in the next period along with capital (no change in the rental rate of capital). Things revert to normal thereafter.

Exercise 16. What happens to labour supply in period 1 if there is a *permanent* rise in productivity?

Solution 16 (Permanent TFP shocks). With permanent shocks, there will only be an income effect (substitution effects are zero) so there will be no intertemporal substitution of labour. People feel richer so work less.