

# Chapter 6

## Econometrics

### 6.1 Introduction

We're going to use a few tools to characterize the time series properties of macro variables. Today, we will take a relatively atheoretical approach to this task, and look to generate some “stylized facts.” A stylized fact is a broad characteristic of the data that is robust across multiple formulations of the problem.

### 6.2 Univariate techniques

#### 6.2.1 Transforming data

The first thing we usually do with many time series variables is transform by taking the natural logarithm. This is because many time series variables have overall trends of exponential growth. Imagine that  $X_t$  is a time series variable with an overall trend growth rate of  $g$ , and deviations from trend with a variance that is proportional to the current level, i.e.,

$$\frac{X_t - X_{t-1}}{X_{t-1}} = g + \epsilon_t$$

where  $\epsilon_t$  is some stationary mean-zero stochastic process. Then  $\ln X_t$  will have a linear time trend with slope  $g$ , and  $\ln X_t - gt$  will be the percent deviation of  $X_t$  from this trend. This suggests we should take logs of variables

we expect to grow exponentially (like GDP per capita, population, consumption, etc.) but not necessarily variables that we expect to fluctuate around a fixed level (like savings rates, interest rates, unemployment rates, etc.). Another reason to take logs is something we have already seen: a Cobb-Douglas production function is linear in logs.

Next we need to consider separating trend from cycle. I'll show you a picture of the time series of log per-capita GDP in the U.S. As we noticed earlier, there is both an overall trend of long-run growth and a series of nontrivial deviations from that trend. So the first step in talking about booms and recessions is to remove the time trend so that we are looking at a stationary process. There are three common ways to detrend a time series  $x_t$ :

- Linear detrend. Run a linear regression of  $x_t$  on  $t$ , and subtract that trend out. Advantage: simple, with well-known properties.
- Differencing filter. Define new variable  $y_t = x_t - x_{t-1}$ . Use  $y_t$ . Notice that if we difference log GDP, we get the growth rate of GDP.
- Other filters. Define a new variable  $y_t$  as some (potentially complex) combination of current and lagged  $x_t$ 's. Most popular is called Hodrick-Prescott filter; some argue for what's called a bandpass filter. Advantage: allows for a flexible trend, i.e., some variation in long-run growth rates (for example allowing for a higher growth rate between 1945 and 1973.).

## 6.2.2 Univariate properties of output

Suppose we have a quarterly series of log per-capita GDP ( $y_t$ ). Let's characterize its time series properties.

First, let's assume that log GDP is "trend stationary", i.e., stationary once a linear trend has been removed, and estimate an AR(1) model.

$$y_t = ay_{t-1} + \epsilon_t \quad (6.1)$$

You can estimate  $a$  by OLS. Typical values found for  $a$  are in the range of 0.9.

This raises an interesting question - 0.9 is very close to one as an estimate, but the difference from a policy perspective is substantial. If output follows a

stationary process, today's recession will be eventually followed by a recovery and output will revert back to its long-run trend. If output follows a random walk (with a deterministic time trend added on), then today's recession will have effects on output far into the future.

During the 1980's there was a great deal of interest in testing to see if output followed an  $I(0)$  or  $I(1)$  process. Romer talks about that debate - the basic conclusion is that maybe output follows an  $I(1)$  process, but there isn't nearly enough data to really know. One application of the research on unit root processes is cointegration.

**Definition 6.2.1 (Cointegration)** *A vector of time series variables  $x_t$  is cointegrated if each element is  $I(1)$ , but there exists a nonzero vector  $a$  (called the cointegrating vector) such that  $ax_t$  is  $I(0)$ .*

The idea is that two variables may be cointegrated if they both bounce around, but there is some long-term "equilibrium" relationship between the two. For example, one might expect that consumption and output are cointegrated. Suppose that:

$$y_t = y_{t-1} + \epsilon_t \quad (6.2)$$

$$c_t = \gamma y_t + u_t \quad (6.3)$$

where  $u_t$  and  $\epsilon_t$  are white noise processes. You would get something like this out of a simple permanent income hypothesis. If you have  $x_t = [c_t y_t]$ , then  $x_t$  is cointegrated with cointegrating vector  $a = [1 - \gamma]$ .

In general the econometric procedure associated with cointegration is:

1. Test for a unit root in each element of  $x_t$ . If they are  $I(1)$ , proceed.
2. Estimate the cointegrating vector (one simple way is by running OLS of one variable on the others).
3. Test for a unit root in  $ax_t$ . If it is  $I(0)$ , the elements of  $x_t$  are cointegrated.

What does it mean to find that a set of variables is cointegrated? Well, it can usually be interpreted as implying that there is a long-run relationship among them.

### 6.2.3 Vector autoregressions

**Definition 6.2.2 (Vector autoregression)** *A vector autoregression (VAR) is simply a regression of the form:*

$$X_t = \Gamma_1 X_{t-1} + \epsilon_t \quad (6.4)$$

where  $X_t$  is a vector of stationary time series and  $\epsilon_t$  is a vector of white noise time series with covariance matrix  $\Omega$ .

Note that the definition I give allows for only a first-order VAR, but our usual trick allows us to put any finite-order VAR into this definition by a simple redefinition of  $X_t$ .

The example we will focus on is money and output:

$$X_t = \begin{bmatrix} y_t \\ m_t \end{bmatrix} \quad (6.5)$$

where  $y_t$  is detrended/differenced output and  $m_t$  is detrended/differenced money supply. We can estimate the  $\Gamma$ 's and the covariance matrix of  $\epsilon_t$  with OLS.

This simple type of VAR has the advantage of characterizing the time series relationship between variables (say money and output) in a nearly atheoretical way. It will turn out to be handy.

## 6.3 Does money matter?

Let's take a look at an example. We really want to know if money has an impact on output.

### Long-run neutrality

First, let's consider long-run neutrality of money. For a very long time, the consensus has been that, even if fluctuations in money affect output in the short term, long-run output is strictly a matter of real (as opposed to nominal) factors.

McCandless and Weber (1995 Minneapolis Fed Quarterly Review) look at average money growth, output growth, and inflation rates for 110 countries over a 30 year time period. They find that the correlation between inflation and the growth rate of the money supply is between 0.92 and 0.96, and that there is no correlation between the output growth rate and either the inflation rate or money growth rate. Other studies find similar results, with some finding a negative correlation and others finding no correlation.

So what does this imply? Well it implies that in the long run the simple quantity theory of money fits well. It also implies that money is neutral in the long run.

### The St. Louis equation

Next we consider whether money is neutral in the short run. The most obvious thing to do is to run a regression of current output on the current money supply (all in log differences or growth rates).

$$\Delta \log(y_t) = b\Delta \log(m_t) + \epsilon_t \quad (6.6)$$

This is often called the “St. Louis equation” because economists from the St. Louis Fed (Andersen and Jordon) used this method in the late 1960’s. We can even stick in some lags of output and money supply. Typical estimates of the St. Louis equation find a positive value for  $b$  - higher money growth is associated with higher output growth.

We could interpret this as saying that money is nonneutral. But there’s a big problem with that. Central bankers watch the economy very closely and adjust monetary policy in response to anticipated output and employment fluctuations.

Suppose that money really has no effect, but that central bankers think it does. Suppose also that output is stationary.

Most central banks will loosen monetary policy when the economy is in the depths of a recession. Because output is stationary, when output is unusually low it will most likely grow more rapidly in the next few quarters. Loose money policy occurs, then growth speeds up. The St. Louis equation would lead us to believe that money matters, when really it does not.

Alternatively, suppose that money was very important to the economy, but that the central bank was very smart and used them to optimally smooth the

economy. If this were the case (and there are some proofs for the following statement), changes in money would be completely uncorrelated with changes in output. So the St. Louis equation can neither prove nor disprove money neutrality.

### A narrative approach

If you've taken some econometrics, you know that the solution to an endogenous explanatory variable is to find an instrumental variable, something that (in this case) affects money growth but has nothing to do with output.

One way to do this is to look for natural experiments, historical events which led to increases in the money growth rate but had nothing to do with output.

I'll describe two examples. The first is from Milton Friedman and Anna Schwartz's *A monetary history of the United States* (1963). They identify numerous examples where U.S. monetary policy changed due to, for example, ideological or personnel shifts within the Fed. The associated movements in the money supply were generally followed by output movements in the same direction. The most famous example is the death of Benjamin Strong in 1928. Strong was the president of the New York Fed with a dominant role in policy setting. After the 1929 stock market crash, the Fed allowed rapid deflation to occur without loosening monetary policy. And in 1931, they engaged in large active tightening of monetary policy. These actions reflected a belief that recessions were necessary to purge the system of weakness. Friedman and Schwartz argue that Strong did not hold such a view and would have led the Fed to pursue an expansionary course (like the Fed after the 1987 crash). Of course, the US economy contracted greatly between 1929 and 1933.

More recently David Romer (the author of your textbook) and Christina Romer followed a similar approach in a 1989 paper. They read the minutes of Fed meetings to identify times when monetary policy was deliberately tightened to reduce inflation (inflation, of course, may have something to do with output, but only if money is non-neutral). In all such cases, the monetary tightening was followed by a recession.

### A structural VAR approach

Another approach to finding monetary policy shocks is to use a “structural VAR”. Notice that in the simple VAR, variables don’t affect each other contemporaneously, and the shocks to output and money are correlated. What we want to know is whether or not an exogenous shock to the money supply leads to an increase in output. Suppose that the structural model is:

$$X_t = B_0 X_t + B_1 X_{t-1} + C u_t \quad (6.7)$$

where  $u_t$  is a vector of *uncorrelated* white noise processes, each with unit variance, and the diagonals of  $B_0$  are zero. The idea here is that there are shocks to money (demand shocks) and direct shocks to output (supply shocks), but that money potentially affects output and output potentially affects money.

We can’t estimate the structural VAR directly. Doing a little algebra, we get:

$$X_t = (I - B_0)^{-1} B_1 X_{t-1} + (I - B_0)^{-1} C u_t \quad (6.8)$$

So if we were to estimate the reduced form VAR, we can recover the structural parameters by solving the system:

$$\Gamma_1 = (I - B_0)^{-1} B_1 \quad (6.9)$$

$$\Omega = (I - B_0)^{-1} C \quad (6.10)$$

The second equation is the one that really matters. We can estimate  $\Omega$ , since it is just the covariance matrix of the reduced form residual  $\epsilon_t$ . Since it’s a covariance matrix and is thus symmetric, if there are  $n$  variables in  $X_t$ , then  $\Omega$  has  $\frac{n^2+n}{2}$  unique elements. Since it has zeros in the diagonal,  $B_0$  has  $n^2 - n$  nonzero elements.  $C$  has  $n^2$  unique elements. We don’t have identification of the structural parameters. Suppose, however, we could use theory to pin down the values of  $\frac{3(n^2-n)}{2}$  parameters. Then we’d be able to recover the rest.

The most common solution is to impose orthogonality of  $C u_t$  (i.e., zeros off of the diagonal of  $C$ ) and a recursive structure on the contemporaneous relationship between the variables. This particular example is from Walsh and Wilcox (1995), which is also described in Carl Walsh’s book *Monetary Theory and Policy*. The VAR has four variables - log CPI, output, log of M2

and the federal funds rate:

$$\begin{bmatrix} CPI_t \\ ICI_t \\ M2_t \\ FF_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ b_{21} & 0 & 0 & 0 \\ b_{31} & b_{32} & 0 & 0 \\ b_{41} & b_{42} & b_{43} & 0 \end{bmatrix} \begin{bmatrix} CPI_t \\ ICI_t \\ M2_t \\ FF_t \end{bmatrix} + B_1 \begin{bmatrix} CPI_{t-1} \\ ICI_{t-1} \\ M2_{t-1} \\ FF_{t-1} \end{bmatrix} + \dots + \begin{bmatrix} \sigma_{vc}v_{C,t} \\ \sigma_{vi}v_{I,t} \\ \sigma_{vm}v_{M,t} \\ \sigma_{vf}v_{F,t} \end{bmatrix} \quad (6.11)$$

As you can see, a number of coefficients in  $B_0$  have been set to zero, enough to identify the rest. This approach is very common - you order the variables so that you can assume that each variable has no contemporaneous effect on the ones above it in the vector, but may have a contemporaneous effect on the ones below.

This allows the authors to estimate the structural parameters. What would you do with these parameters?

One approach is to plot impulse response functions (see handout, which depicts figure 1.4 from Walsh).

Another is to perform variance decompositions (see below). The idea of a variance decomposition is that the variance in  $k$ -step-ahead forecast error ( $E[(x_t - E_{t-k}x_t)^2]$ ) is a linear combination of the variance of each of the orthogonal shocks. As a result, you can divide this forecast error into parts which are attributable to each of the shocks.

Months after shock	CPI	ICI	M2	FF
0	0.3	99.7	0	0
6	3.7	76.9	7.6	11.9
12	25.1	49.6	9.8	15.5
18	32.6	33.1	9.2	25.1
24	27.7	26.0	18.9	27.4
36	20.4	23.5	39.5	16.6

So what has all of this added up to? There is a consensus that:

- Money is neutral in the long run.
- Money is non-neutral at business cycle frequencies.
- Though it does not explain a large portion of output fluctuations, there is evidence that shocks to money have an output effect and that monetary policy has played an important role in several recessions.