Income stream Y_1, Y_2. These are *endowments*

$$u(C_1, C_2) = u(C_1) + \beta u(C_2)$$

$$C_1 + S = Y_1$$

$$C_2 = Y_2 + (1 + r)S$$

(Can also explicitly model bonds)

Intertemporal Budget Constraint:

$$C_1 + \frac{C_2}{1 + r} = Y_1 + \frac{Y_2}{1 + r}$$

Take r as given and maximize. Two equations, 2 unknowns (C_1 and C_2). Get Euler equation:

$$u'(C_1) = \beta(1 + r)u'(C_2)$$

Intuition (MC=MR).

Give up one unit of consumption today: lose $u'(C_1)$ today.

Receive $1+r$ units of consumption tomorrow (r is a real interest rate). In utility terms, you receive $(1+r)u'(C_2)$ tomorrow.

But because of discounting, you value this by only $\beta(1+r)u'(C_2)$.

If we are at an optimum, we can't raise utility any further by doing this rearranging and hence

$$u'(C_1) = \beta(1+r)u'(C_2)$$

In a multi-period model, this relationship holds for *any* two consecutive periods. Therefore, it nails down the entire lifetime consumption profile.

E.g., Log utility

$$\frac{C_2}{C_1} = \beta(1+r)$$

Combine with budget constraint to get

$$C_1(1+\beta) = Y_1 + \frac{Y_2}{1+r}$$

$$C_1 = \frac{1}{1+\beta}\left(Y_1 + \frac{Y_2}{1+r}\right)$$

Notes

- C today depends on lifetime resources; naturally extends to more periods. Contrast with $C = C_0 + \beta Y$.

- We could also model wealth effects. If receive bequest of A next period, then

$$C_1 = \frac{1}{1+\beta}\left(Y_1 + \frac{Y_2}{1+r} + \frac{A}{1+r}\right)$$

- Precautionary savings. A given amount of expected income has a lower *certainty equivalent* level of income. Hence, uncertainty acts like a fall in future income and reduces consumption today, giving rise to *precautionary savings*. As an example, suppose income this period is 5 and next period is *either* 0 or 10 with equal likelihood. Thus expected income next period is 5. With precautionary savings, you will value this *expected* (yet uncertain) income less; say, it might be worth only 3 to you (in terms of *certain* income.) In this case, you will consume only $\frac{5+3}{2} = 4$ in period one. Intuitively, you are very concerned about the 0 eventuality next period, and save more *today* as a precaution.

Aside on standard utility, $u(C) = \frac{C^{1-\theta}}{1-\theta}$.

$u'(C) = \frac{1}{C^\theta}$: θ governs degree of DMU.

$\frac{1}{\theta}$ is IES.

High θ implies person is satiated quickly; this induces a weak substitution effect. Think of salt: if all consumption goods were like salt, θ would be very high. Indeed, θ is relatively high in the data, implying people seek to eagerly smooth consumption over time. As a result, response to interest rate changes is low.

Labour/Leisure Choice

Start with Static Model

$\max_{c,l} u(c) - v(l)$ subject to $c = wl + d$.

Note that $u'' < 0$ and $v' > 0$.

$$u'(c)\frac{dc}{dl} - v'(l) = 0 \Longrightarrow u'(c)w = v'(l)$$

- Form of $v(l)$. Labour-smoothing

- General Equilibrium ($w = MPL$)

- Can have corner solution

$$wu'(c) < v'(l) \Longrightarrow l^* = 0$$

Participation (extensive margin). E.g., high unemployment benefits, would raise d and reduce marginal utilty – and can thereby induce non-participation. Effect would depend on size of d and on θ from the utility function; how quickly do you become satiated?

Suppose $u(C) = \frac{C^{1-\theta}}{1-\theta}$, $v(l) = \frac{1}{2}l^2$, and $c = wl$. The consumer solves

$$\max_l \frac{(wl)^{1-\theta}}{1-\theta} - \frac{1}{2}l^2$$

The first-order condition is:

$$\frac{w}{(wl)^{\theta}} = l \Longrightarrow l^* = w^{\frac{1-\theta}{1+\theta}}$$

If $\theta > 1$, $\frac{dl}{dw} < 0$, and the income effect dominates.

If $\theta < 1$, $\frac{dl}{dw} > 0$.

Empirical tests? Adjustment costs.

Multi-period model

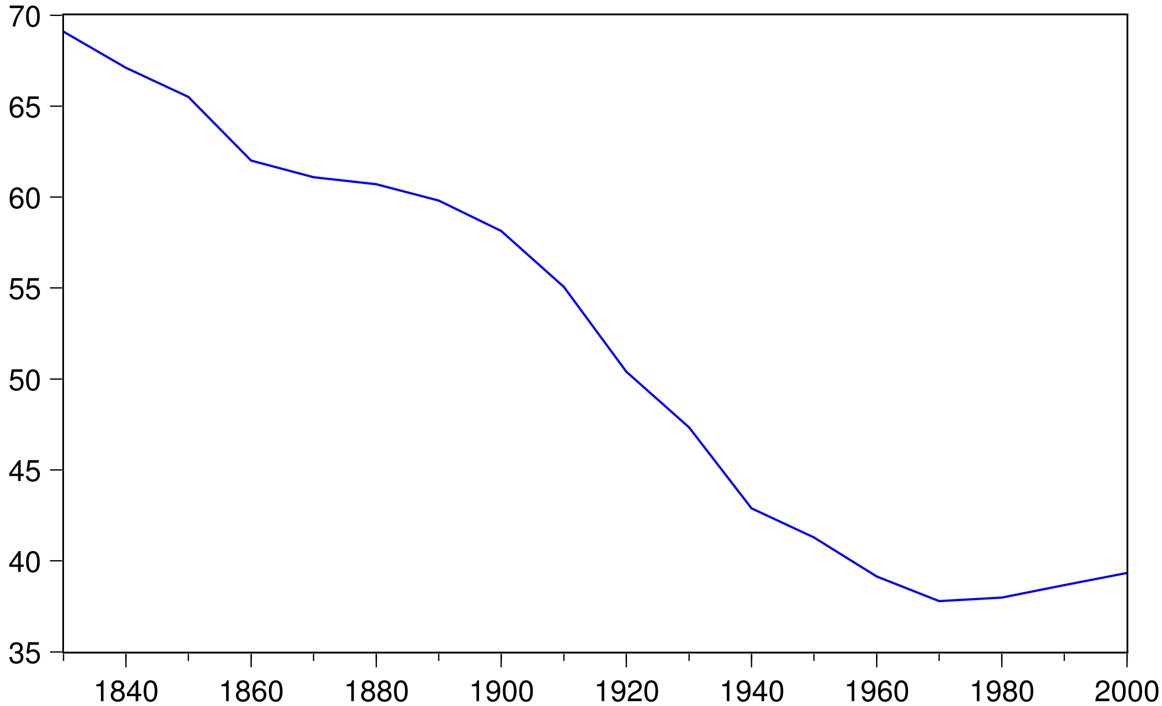
$$wu'(c) = v'(l)$$

Using standard utility, $u(C) = \frac{C^{1-\theta}}{1-\theta}$, and $v(l) = \frac{1}{2}l^2$, we have

$$\frac{w}{C^\theta} = l$$

Permanent Changes (Long-run Trends): By the permanent income hypothesis, if w rises permanently, C should rise permanently too. Over time, therefore, w and C grow at around the same rate, so if $\theta > 1$, labour supply falls over time. The fact that labour supply doesn't rise over time is strong evidence against $\theta < 1$.

Temporary Changes: With a temporary change, w rises and C only rises a little (since consumers smooth the temporary rise in income over time.) As a result, l rises temporarily. This mechanism is important over the business cycle (where wages are mildly procyclical.)



Weekly Labor Hours: U.S., 1830-2000

Two period model.

$$u(C_1) - v(l_1) + \beta(u(C_2) - v(l_2))$$

$$C_1 + \frac{C_2}{1+r} = wl_1 + \frac{wl_2}{1+r}$$

The optimality conditions become:

$$u'(C_1) = \beta u'(C_2)(1+r)$$

$$wu'(C_1) = v'(l_1)$$

$$wu'(C_2) = v'(l_2)$$

Reading: Romer's chapter on Fiscal Policy.

Example 1: A lump-sum tax (e.g., a property tax)

$$u(C_1) - v(l1) + \beta(u(C_2) - v(l_2))$$

$$C_1 + \frac{C_2}{1+r} = wl_1 + \frac{wl_2}{1+r} - T_1 - \frac{T_2}{1+r}$$

The optimality conditions become:

$$u'(C_1) = \beta u'(C_2)(1+r)$$

$$wu'(C_1) = v'(l_1)$$

$$wu'(C_2) = v'(l_2)$$

Because income falls, the consumer is poorer; as a result, demand for consumption and *leisure* will fall. For this reason, labour supply rises – a *pure income effect*. Relative prices are not distorted making this tax more efficient.

E.g., Labour supply of old during recession. Likewise, lottery winners reduce their labour supply.

Example 2: A rise in the tax rate on labour.

$$u(C_1) - v(l_1) + \beta(u(C_2) - v(l_2))$$

$$C_1 + \frac{C_2}{1+r} = (1-t)wl_1 + \frac{(1-t)wl_2}{1+r}$$

The optimality conditions become:

$$u'(C_1) = \beta u'(C_2)(1+r)$$

$$(1-t)wu'(C_1) = v'(l_1)$$

$$(1-t)wu'(C_2) = v'(l_2)$$

Note that because of consumption and labour smoothing (same tax rates), the C and l variables will move together.

If t rises, what happens? The budget constraint and first-order conditions must be satisfied.

This depends on the interaction of the income and substitution effects. Which dominates? Long-run evidence suggests income effect dominates, in which case labour supply would rise.

More formally and looking at the budget constraint and first order conditions:

- C could rise and l could fall (but this contradicts budget constraint).

- C could rise and l could rise (but this contradicts labour condition).

- C could fall and l could fall. C could fall and l could rise. The latter are possible, but it's unclear which one. Formally, there are income and substitution effects, and either could dominate. What happens depends on the functional form for utility.

Note that we could also have a tax on consumption (e.g., VAT) or interest income.

Example 3: Higher tax rate, but revenue given back. Edward Prescott story.

$$u(C_1) - v(l_1) + \beta(u(C_2) - v(l_2))$$

$$C_1 + \frac{C_2}{1+r} = (1-t)wl_1 + \frac{(1-t)wl_2}{1+r} + twl_1 + \frac{twl_2}{1+r}$$

$$\Longrightarrow C_1 + \frac{C_2}{1+r} = wl_1 + \frac{wl_2}{1+r}$$

The optimality conditions become:

$$u'(C_1) = \beta u'(C_2)(1+r)$$

$$(1-t)wu'(C_1) = v'(l_1)$$

$$(1-t)wu'(C_2) = v'(l_2)$$

Revenue Neutrality: the government redistributes the revenues. Overall, wages are lower, but income remains the same. So there is a *pure substitution* effect and no income effect. Labour supply falls unambiguously. This can explain the negative relationship between tax rates and labour supply in the data.