Problem Set 1: Consumption Theory & Labour Supply

Consumption Theory & Labour Supply

Exercise 1. Consider a one-period model where the only income source is labour. Suppose the utility function of the representative agent is $log(c) - \frac{l^2}{2}$, and there is a constant tax rate t on labour supply:

1. What is the optimal level of labour supply?

2. What revenue is raised (call this T)?

3. If a lump-sum tax of T was imposed on the worker, what would happen to labour supply? (Use intuition.)

Exercise 2. Write down the labour-leisure optimality condition in terms of the real wage, $\frac{W}{P}$. Assuming people derive all their income from labour income, show that a sales tax has the same economic effects on labour supply as a tax on labour. Explain the intuition.

Exercise 3. Suppose we have a two period model, and each period utility is given by

$$u(C) = \frac{C^{1-\theta}}{1-\theta} - \frac{L^{1+\sigma}}{1+\sigma}$$

Assuming $\beta(1+r) = 1$ show that

$$\frac{l_1}{l_2} = \left(\frac{w_1}{w_2}\right)^{\frac{1}{\sigma}}$$

Explain the intuition (esp. the role of σ), and use the condition to explain what happened in Iceland in 1987.

Exercise 4. Suppose the utility function is

$$u(c,l) = \log(c) - \gamma\frac{l^{1+\sigma}}{1+\sigma}$$

The real wage is w. Assume consumption is financed through labour income.

a) What role does σ play?

b) Find the optimal level of labour supply.

c) What is the interpretation of γ? Could differences in labour supply across countries be explained by differences in γ?

d) Suppose now utility takes the more general form $u(c,l) = \frac{c^{1-\theta}}{1-\theta} - \gamma\frac{l^{1+\sigma}}{1+\sigma}$ and $\theta = \frac{1}{2}$. Find the optimal level of labour supply. What happens to labour supply if the government imposes a proportional tax on the real wage?

e) Explain what happens to labour supply if the government gives people a "handout" of d. (No need for maths.)

Exercise 5. Suppose lifetime utility is given by

$$u(C_1, l_1, C_2, l_2) = \log(C_1) - .5l_1^2 + \log(C_2) - .5l_2^2$$

The real interest rate is r and wages in periods 1 and 2 are w_1 and w_2, respectively. Write down the intertemporal budget constraint, assuming the government imposes a lump sum tax of T each period. Write down the first order conditions for labour and indicate how labour supply is affected by this change.

Exercise 6. Consider the *Ricardian* approach to fiscal policy, but now assume endogenous income (via labour supply). If government expenditure rose permanently, what would happen to i) consumption; ii) labour supply?; and iii) output? With "GHH" preferences, the marginal utility of consumption is increasing in the level of labour supply (i.e., consumption and labour are complements in utility). How would this change the results?

Exercise 7. Suppose there are three tax brackets in a progressive tax system. The marginal tax rate is 10% up to 10,000, 20% from 10,000 to 20,000, and 30% above 20,000. Suppose the middle rate is reduced from 20% to 10%.

1. What effect will this have on the labour supply of a worker earning i) 10,000; ii) 20,000; and iii) 30,000?

2. What is the maximum tax revenue from a worker earning 30,000 now?

3. Explain why social welfare can be improved by reducing the marginal tax rate of the richest worker in the economy to zero. Does this logic hold for the *second* richest person in the economy?

Exercise 8. In year 1, suppose that there is a temporary increase in government expenditure (due, say, to a war) in country A. Meanwhile, in country B there is a permanent rise in expenditure due to an expansion of the welfare state. According to the theory of *tax smoothing*, what happens to the government's *bond issuance* in each country in year 1?

Exercise 9. Write down the undistorted labour/leisure in terms of the real wage, $\frac{W}{P}$, and assume optimal labour supply is l^*. Suppose now there is a proportional sales tax of rate τ on all goods. To ensure labour supply of l^*, at what rate would the government have to *subsidise* nominal wages?

Exercise 10. Multiple Choice Questions: 2012 Midterm, Questions 1-7 & 12.