Problem Set 2: Labour, General Equilibrium & Monetary Policy

Labour & General Equilibrium

Exercise 1. Some people claim that the U.S. labour market remains depressed since people expect taxes on labour income to be higher in the future. Is this view consistent with the *intertemporal substitution of labour*?

Solution 1 (Intertemporal Substitution of Labour)**.** This view is inconsistent with the intertemporal substitution of labour. If people expect taxes on labour income to be higher in the future, then they perceive today's after-tax income to be higher than future after-tax income so that the substitution effect would induce them to work more today. (However, looking at the income effect: they earn less in the future so will be poorer then so work more then). Seeing that they could earn more today relative to the future, they should work more today and the evidence of not working as much today is a sign that they do not substitute labour intertemporally too much.

Exercise 2. To maximize workers' welfare, why is it optimal to have a zero tax rate on capital, but a 100% tax rate on all *existing* capital?

Solution 2 (Capital Taxation)**.** Existing capital is illiquid so can be taxed efficiently. Taxing capital that does not yet exist discourages investment and without sufficient capital, worker's marginal productivity of labour and their real wages will not be as high.

Exercise 3. Suppose a person lives for T periods and their current and future real income increases *permanently* by Y. According to the permanent income hypothesis, how should consumption respond? What would happen if higher income uncertainty is associated with higher incomes? (Assume marginal utility is strictly convex).

Solution 3 (Precautionary Savings)**.** According to the permanent income hypothesis, consumption should rise one-for-one. However, the attendant uncertainty would induce a rise in *precautionary* savings, which would reduce today's consumption response. Overall, therefore, consumption would not rise one-for-one.

Exercise 4. Suppose that there are two closed economies – A and B – in the world. Except for the fact that consumers in country A have a higher discount factor (β) than in country B, both countries are identical. Which country has the higher natural interest rate? If A has a higher θ than B, where does the interest rate rise most if investment demand rises?

Solution 4 (Long-Run General Equilibrium Model)**.** From $u(C_1, C_2) = u(C_1) + \beta u(C_2)$, we know that residents of a country with a higher discount factor β will place more weight on the future. As a result, the residents of country A will save more, which will lead to a lower natural interest rate in A in equilibrium. Another way of looking at this is as follows. Higher rates of time preference (lower discount rate, β) indicate unwillingness to save and so the natural rate of interest has to be higher. Country B has the lower β so it has the higher natural interest rate. $\frac{1}{\theta}$ measures the intertemporal elasticity of substitution and country B has a higher $\frac{1}{\theta}$ since country A has a higher θ. Country A will need to be rewarded in terms of higher interest rates if investment demand rises.

Exercise 5. Suppose that, in a particular country, there is a high mortality rate. You could represent this as a probability p that the consumer will not exist the following period. Using the stochastic Euler equation, determine how this would affect the natural rate of interest and the level of equilibrium investment each period.

Solution 5 (Stochastic Euler Equation)**.** Suppose that the only uncertainty arises from whether a person is alive or dead next period. From the standard model, we have lifetime utility of $u(C_1) + \beta E u(C_2)$. The stochastic Euler equation is $u'(C_1) = \beta E(1+r)u'(C_2)$. Then using the definition of expectation and assuming the marginal utility when dead is 0, this reduces to $u'(C_1) = \beta p(0) + \beta(1-p)(1+r)u'(C_2) = \beta(1-p)(1+r)u'(C_2)$. As a result, we get a lower discount factor; this leads to lower savings, a higher natural interest rate and lower investment in equilibrium. Technically, $\frac{u'(C_1)}{u'(C_2)} = \beta(1-p)(1+r) < \beta(1+r)$. Because the ratio $\frac{u'(C_1)}{u'(C_2)}$ is now lower, C_1 must be higher for a given income stream.

Exercise 6. From 1982-84, long-run interest rates rose significantly in Japan. Explain how this was caused by rising U.S. budget deficits. Explain too how the latter development led to the emergence of so-called "twin deficits": the simultaneous occurrence of budget *and* current account deficits.

Solution 6 (Twin Deficits)**.** The rise in U.S. budget deficits leads to a fall in national U.S. savings, placing upward pressure on U.S. interest rates. This will attract capital inflows from other countries, including Japan. These outflows from *those* countries will raise interest rates in those countries. Meanwhile, the inflows of capital to the U.S. will strengthen the U.S. exchange rate, thereby reducing export demand and raising import demand. For these reasons, the U.S. current account balance deteriorates.

Monetary Policy

Exercise 7. Paul Krugman has argued that that natural rate of interest is currently *negative*. If the nominal rate is zero, how could this be attained?

Solution 7 (Negative Interest Rates)**.** From the Fisher equation, $i = r + \pi$, the natural rate is $r_n = i - \pi = 0 - \pi$. For a given nominal rate of 0, we can attain a negative real rate through raising inflation; i.e., money growth.

Exercise 8. Write down the *Euler equation* between periods 1 and 2. Write the Euler equation for periods 2 and 3. Use the *expectations theory of the term structure* to write the Euler equation between periods 1 and 3, in terms of the interest rate on a 2-year bond. Assume no risk premium, no inflation, and no uncertainty.

Solution 8 (Expectations Theory & Euler Equation)**.** Assuming short-run rates of r_1 and r_2 between periods 1 and 2 and 2 and 3 respectively, the Euler equations are $u'(C_1) = \beta(1+r_1)u'(C_2)$ and $u'(C_2) = \beta(1+r_2)u'(C_3)$. Combining yields $u'(C_1) = \beta^2(1+r_1)(1+r_2)u'(C_3)$. But according to the expectations theory, $(1+r_1)(1+r_2) = (1+r_{2l})^2$, where r_{2l} denotes the long-run rate on the 2-year bond. As a result, $u'(C_1) = \beta^2(1+r_1)(1+r_2)u'(C_3) = \beta^2(1+r_{2l})^2u'(C_3)$. Overall, $u'(C_1) = \beta^2(1+r_{2l})^2u'(C_3)$.

Exercise 9. If the nominal interest rate on a two year bond is 6%, and the current short-run interest rate is 2%, what are expectations of the short-run rate for next year? [Assume no risk premium in long-run rates].

Solution 9 (Expectations Theory)**.** According to the expectations theory (with no risk/term premium), $i_{2l} = \frac{i_1 + Ei_2}{2} = \frac{2+Ei_2}{2} = 6$. This implies $Ei_2 = 10\%$.

Exercise 10. Suppose the central bank raises the rate of money growth *permanently* from 0% to 2%. The current short-run nominal interest rate is 4%. Assume prices are fully flexible. What would happen to long-run rates? What effect would central bank independence have on long-run rates?

Solution 10 (Central Bank Independence)**.** According to the expectations theory, long-run rates would rise as future expected short-run rates would rise as inflation would rise by 2% since prices are fully flexible (using the quantity theory: $\pi = g_m - g_y$ where g_m and g_y are money growth and income growth, respectively). Consequently, the yield curve would shift upwards uniformly right across the term structure. Central bank independence would dampen the increase on long-run rates, so that low short-run rates could translate into low long-run rates, which are the important ones to stimulate the economy.

Exercise 11. If the natural real interest rate is 4%, what is the natural *nominal* interest rate? Assume money growth is 2% percent, and output growth is 3%.

Solution 11 (Quantity Theory of Money & Fisher Equation)**.** From the quantity theory, $\pi = g_m - g_y = 2\% - 3\% = -1\%$. From the Fisher equation, $i_n = r_n + \pi \implies i_n = 4\% - 1\% = 3\%$.

Exercise 12. Using the expectations theory of the term structure, explain the following:

> 'Reports that the fiscal stimulus package could total 600 billion over 10 years, much larger than expected by bond investors contributed to a further sell-off yesterday among concerns about rising future issuance of government bonds. Yesterday, five-year and 10-year yields ended at 3.04% and 4.06%, respectively, up from 2.98% and 4.03% on Friday.'

Solution 12 (Expectations Theory)**.** The rise in future deficits would cause expectations of the natural rate to rise in the future. According to the expectations theory, this would cause long-run rates to rise *today*.

Exercise 13. Multiple Choice Questions: 2012 Midterm, Questions 8-10; 2011 Midterm, Questions 1-9.

Solution 13 (MCQs)**.** 2012: e, d, f. 2011: b, e, d, d, c, a, c, c, a.