

More on Behind Testing Monetary Neutrality

Michael Curran

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1 Econometrics

1.1 Detrending

Advanced reading for those interested: (i) Chapter 6 (especially page 119 and section 6.2.5-7) of DeJong & Dave ‘Structural Macroeconometrics’; (ii) pages 208-211 of MSc-notes.pdf at <http://www.michael-curran.com/teaching/MSc-notes.pdf>

To focus on cycles say, rather than long trends, i.e. if we are interested in looking at movements in the data that occur in the short or medium term rather than in long-run growth, we need to ‘detrend’ or filter the data. The Hodrick-Prescott filter is the most widely used filter, followed in second place by the Bandpass filter. Think about the following like you are adjusting the sound on your sound system / i-Phone / car radio, etc. By specifying parameters within the Hodrick-Prescott filter, we can ‘turn up the volume’ on particular frequencies (bass / treble, etc.) – we can focus on short (high-frequency) cycles of a few months or medium cycles such as those at business cycle frequencies (approximately 8 years) or we can even simply look at long-run, low frequency cycles (more than 8 years). The bandpass filter essentially shuts down all fluctuations *outside* of a chosen frequency band, e.g. we could just focus on cycles with periods between 6 and 40 quarters as in business cycle applications.

Decomposing $\log y_t$ into a growth component g_t and a cyclical component c_t , the Hodrick-Prescott (H-P) filter estimates g_t and c_t in order to minimise

$$\sum_{t=1}^T c_t^2 + \lambda \sum_{t=3}^T [(1-L)^2 g_t]^2$$

taking λ as given, where L is the *lag operator*, i.e. $Lg_t = g_{t-1}$, $L^2 g_t = g_{t-2}$. Trend removal is accomplished simply as

$$\tilde{y}_t = \log y_t - \hat{g}_t = \hat{c}_t$$

The parameter λ determines the importance of having a smoothly evolving growth component: the smoother is g_t , the smaller will be its second difference. With $\lambda = 0$, smoothness receives no value and all variation in $\log y_t$ will be

assigned to the trend component. As $\lambda \rightarrow \infty$, the trend is assigned to be maximally smooth, i.e. linear. Generally, λ is specified to strike a compromise between these two extremes. In working with business cycle data, the standard choice is $\lambda = 1600$ (quarterly data).

1.2 Stationarity

- Keep in mind the concept of **stationarity** (mean and variance are constant over time, i.e. $E(\epsilon_t) = \mu, V(\epsilon_t) = \sigma^2$). If one or both of these two conditions are not met, then the series (e.g. GDP) is non-stationary.
- In the notes, it says that a **Cobb-Douglas production function is linear in logs**. To see this, note that a Cobb-Douglas production function is given by

$$Y(K, L) = AK^\alpha L^{1-\alpha}$$

So taking logarithms and observing that the logarithm of a product is the sum of logarithms and the logarithm of X to the power of b is simply b times the logarithm of X :

$$\log(Y(K, L)) = \log(A) + \alpha \log(K) + (1 - \alpha) \log(L)$$

Note when using Ordinary Least Square regression (OLS) on an AR(1) model (see section 6.2.2 in notes on econometrics / testing money neutrality), the resulting estimates will be biased (the expected mean of the estimate will not equal that of the parameter you are trying to estimate) but consistent (as you increase the sample size, the distribution of the estimate will converge to the actual parameter you are trying to estimate).

An I(1) series, e.g. a random walk process $y_t = y_{t-1} + \epsilon_t$ is ‘integrated of order 1’ (hence ‘I(1)’) since we can take differences once (‘first differences’) to render it stationary: y_t is I(1) but subtracting last period from the current period (first differences) $y_t - y_{t-1} = \epsilon_t$ and ϵ_t is stationary (for instance, it could be *white noise*, i.e. $\epsilon_t \sim N(0, \sigma^2)$: constant *zero* mean and constant variance). An I(1) series is also called ‘difference stationary’ since first differencing renders such a series stationary. An I(0) series is stationary by definition: we don’t need to take any differences (zero) to render the series stationary. The 1980s were the haydays for econometricians looking at problems when you have ‘unit roots’ (I(1) processes – yes there are lots of terms in time series!) or close to unit roots, e.g.

$$y_t = ay_{t-1} + \epsilon_t$$

where estimates for a are close to or equal to one. If indeed a is one, then policy is REALLY important since today’s recession (y_t drops) is extremely persistent!

Two series are said to be *cointegrated* if each element is I(1) but some linear combination of the series is I(0). The typical example given is the drunk owner with a dog on a leash walking around Hyde Park. The owner walks around pretty randomly, though the exact position of the owner in the park in any given

second is very closely related to his position immediately beforehand ($y_t = y_{t-1}$) plus a random term (ϵ_t) since he is drunkenly wandering. Since the dog is on a leash following the owner around, the dog's position is also well described by this $I(1)$ process. However, the dog never gets too far away from the owner, so there is a long run relationship: in terms of position: $owner_t - dog_t$ is actually $I(0)$ even though each is $I(1)$. This is similar for comparing consumption c_t with GDP y_t : each are $I(1)$, but there is a combination $c_t + a \cdot y_t$ that is $I(0)$, e.g. when $a = -1$. In the notes on testing monetary neutrality

$$\begin{aligned}y_t &= y_{t-1} + \epsilon_t \\c_t &= \gamma y_t + u_t\end{aligned}$$

where ϵ_t and u_t are white noise processes (so $I(0)$).

$$x_t = \begin{bmatrix} c_t \\ y_t \end{bmatrix} \quad a = \begin{bmatrix} 1 & -\gamma \end{bmatrix}$$

While $c_t \sim I(1)$ and $y_t \sim I(1)$

$$cx_t = c_t - \gamma y_t = \gamma y_t + u_t - \gamma y_t = u_t \sim I(0)$$

1.3 VAR

Note that $\text{VAR}(1)$ means we have Γ_1 . $\text{VAR}(p)$ would be where we have $\Gamma_1, \Gamma_2, \dots, \Gamma_p$. The notes mention that ‘our usual trick allows us to put any finite-order VAR into this definition by a simple redefinition of X_t .’ By finite-order, they mean $p < \infty$. The trick is to define the matrix X_t as say $X_t = (y_t, y_{t-1}, y_{t-2}, \dots, y_{t-p+1})$ for instance, so we could still use $\text{VAR}(1)$ but have lags going back further than one period, so we don't need to have more Γ terms and an ugly $X_t = \Gamma_1 X_{t-1} + \Gamma_2 X_{t-2} + \dots$

2 Narrative Approach

- Note that the *Volker Recession* was caused by a large reduction in the money supply (mainly from the inflation aversion of Paul Volcker, the then FED chairman) followed by a large recession.
- Devaluations: countries that devalue – and emerge from monetary straight-jackets – typically see large rises in output. E.g. countries that left the gold standard during the Great Depression recovered quickly (and in the order in which they left).