Representative Firm and Household.

$$E_0 \sum_{t=0}^{t=\infty} \beta^t \left(\log C_t - \frac{l_t^{1+\sigma}}{1+\sigma} \right)$$

$$w_t l + r_t k_t = C_t + i_t$$

$$k_{t+1} = (1-\delta)k_t + i_t$$

No profits in budget constraint (since firms are perfectly competitive).

Combining

$$w_t l_t + r_t k_t = C_t + k_{t+1} - (1-\delta)k_t$$

$$w_t l_t + r_t k_t = C_t + k_{t+1} - k_t + \delta k_t$$

$$w_t l_t + (1 + r_t - \delta) k_t = C_t + k_{t+1}$$

$$\underbrace{w_t l_t + (1 + r_t - \delta) k_t}_{sources} = \underbrace{C_t + k_{t+1}}_{destinations}$$

Tradeoff.

Time constraint.

Set $\delta = 0$ (or think of r_t as real return *net of depreciation*). For any two consecutive periods, we have

$$u'(C_t) = E_t \beta (1 + r_{t+1}) u'(C_{t+1})$$

Trajectory of A is uncertain, but we know its expectation and variance. As a result, r_{t+1} is uncertain; hence we need an expectation sign in Euler equation.

Note that r_{t+1} is "interest rate" in Euler equation: buy capital today to rent out *next* period.

Labour optimality condition:

$$w_t u(C_t) = v'(l_t)$$

No expectation operator here, since all variables are in same period.

Implicitly, gives labour supply

$$w_{t+1} u(C_{t+1}) = v'(l_{t+1})$$

$$\lim_{t \longrightarrow \infty} \beta^t u'(C_t) k_t = 0$$

$$\beta = \frac{1}{1+r}$$

Like before, w and r are endogenous (a general equilibrium model).

Will derive labour and capital demand functions in solving the firm's problem.

Then, we will equate supply and demand for each market.

Since $u(C_t) = \log C_t$

$$\frac{w_t}{C_t} = l_t^\sigma$$

$$\frac{w_{t+1}}{C_{t+1}} = l_{t+1}^\sigma$$

$$\frac{1}{C_t} = E_t \beta(1 + r_{t+1}) \frac{1}{C_{t+1}}$$

For simplicity, set $\beta(1 + r_{t+1}) = 1$ and ignore uncertainty:

$$\frac{l_{t+1}^\sigma}{l_t^\sigma} = \frac{w_{t+1}}{w_t}$$

$$\Longrightarrow \frac{l_{t+1}}{l_t} = \left(\frac{w_{t+1}}{w_t}\right)^{\frac{1}{\sigma}}$$

This represents the *intertemporal substitution of labour*. This is a key *propagation mechanism* in this model.

More generally, when $\beta(1 + r_{t+1}) \neq 1$ and no uncertainty:

$$\frac{l_{t+1}^{\sigma}}{l_t^{\sigma}} = \frac{w_{t+1}}{w_t} \frac{1}{\beta(1 + r_{t+1})}$$

$$\Longrightarrow \frac{l_{t+1}}{l_t} = \left(\frac{w_{t+1}}{w_t} \frac{1}{\beta(1 + r_{t+1})} \right)^{\frac{1}{\sigma}}$$

Important point: degree of intertemporal substitution depends on σ.

RBC'ers must argue σ is relatively low for intertemporal substitution to be important.

The Representative Firm.

Price taker, perfect competition. For simplicity, ignore time subscripts here.

$$\pi = AK^{\alpha}L^{1-\alpha} - wL - rK$$

$$(1-\alpha)AK^{\alpha}L^{-\alpha} - w = 0$$

First order conditions:

$$\frac{\partial \pi}{\partial L} = (1-\alpha)AK^{\alpha}L^{-\alpha} - w = 0$$

Above condition gives labour demand.

$$\frac{\partial \pi}{\partial K} = \alpha AK^{\alpha-1}L^{1-\alpha} - r = 0$$

Above condition gives capital demand.

Solve for L and K in first order conditions above to get labour and capital demands.

Key point: *Labour and Capital demand are both increasing in* A.

More importantly, wages and rental rates are increasing in A.

General equilibrium (Combine HH and Firm).

What increases MPK and MPL (like before).

Major point; wages and interest rates depend on A and capital stock.

$$\frac{\partial F}{\partial L}L + \frac{\partial F}{\partial K}K = wL + rK = Y$$

CRS; zero profits.

Labour productivity.

Solow residual is *exogenous* and *induces* movements in Y.

NK response: Solow residual is *endogenous* to cycle.

Think of restaurant and labour hoarding.

Labour hoarding, IRS, varying capital/labour utilization. Keynesians say production function looks like

$$Y_t = A(uK)^{\alpha}(uL)^{1-\alpha} = AuK^{\alpha}L^{1-\alpha}$$

where u denotes *utilization* rate or effort (say), and is *endogenous* to cycle. High u is equivalent to high A. According to Keynesians, what looks like changes in A are really changes in u.