Problem Set 5: New Keynesian Model

New Keynesian Model

Exercise 1. In 'The Ends of Four Big Inflations', Thomas Sargent observed how large hyperinflations ended without recessions. How could this be reconciled with the New Keynesian Phillips Curve?

Solution 1 (Hyperinflation & New Keynesian Phillips Curve)**.** Hyperinflation usually refers to very high (and usually accelerating rates of) inflation. The New Keynesian Phillips Curve is $\pi_t = \frac{\alpha\delta}{1-\delta}(y_t - y_n) + \phi E_t \pi_{t+1}$. For π_t to come down, either both terms decline or one declines more than the other increases (stays the same). Since with hyperinflation, people expect high future inflation, the first term is more likely to come down. Rather than $y_t - y_n$ falling, $\frac{\alpha\delta}{1-\delta}$ could fall. Either δ falls or α falls or both or the decline in $\frac{\delta}{1-\delta}$ is greater than the rise in α (or *vice-versa*). A fall in δ means that firms have a lower chance of changing their prices or equivalently, a lower fraction of firms change their prices this period (so inflation won't change by as much since firms are changing prices less often (then there will be less inflation pressure for any given output gap)). A lower α means that marginal costs (and hence prices) react less to changes in the output gap, i.e. there is more real rigidity and/or nominal rigidity (the latter since prices don't change much as output varies over the business cycle). The extent to which wages – and hence marginal costs – fall depends on factors like the elasticity of labour supply; hence, α depends on features of the economy.

Exercise 2. If prices are flexible in the New Keynesian model, then money is neutral, yet output is inefficiently low. Is this statement true or false?

Solution 2 (New Keynesian Model)**.** The statement is true. If we introduced money, the usual money market equilibrium condition must be satisfied in equilibrium (e.g money in utility function or cash-in-advance constraint and derive money demand from microfoundations): $\frac{M}{P} = L(r_n, Y_n)$. But Y_n and r_n are already determined. In this model, if prices are flexible, a rise in M just rises P and money is still neutral. Notice that P must adjust since r_n and Y_n have already been nailed down by fundamentals. Another way of seeing money neutrality is as follows. With flexible prices, marginal costs rise in booms as real wages can rise to meet extra labour demand. There is less real rigidity, reducing the effectiveness of monetary policy. Money is neutral when prices are flexible. Regarding the efficiency of output, note that in flexible price equilibrium, the optimal price a firm charges still is $P_i = \frac{\eta}{\eta-1}MC$ where $\frac{\eta}{\eta-1}$ is the target markup. Because of monopolistic competition – and so $P > MC$ – output will be below the socially efficient level.

Exercise 3. Intuitively, how would a low marginal disutility of labour supply affect the coefficient on the output gap in the NKPC? What are the implications of this for the effectiveness of monetary policy?

Solution 3 (Marginal Disutility of Labour Supply)**.** A low marginal disutility of labour supply would mean that people would be somewhat indifferent to working, say, 100 hours one day and 50 hours a day for two days. So, people would not need to be compensated as much in terms of real wage increases to work more. Therefore, marginal costs would not rise by as much in response to the output gap increasing, i.e. there would be more real rigidity and the parameter α that measures the responsiveness of marginal costs to the output gap would decrease. α is part of the coefficient on the output gap in the New Keynesian Phillips Curve $\frac{\alpha\delta}{1-\delta}$ and δ (proportion of firms changing prices each period) does not change so the coefficient on the output gap in the NKPC would decrease. Intuitively, a low α makes marginal cost less sensitive to output gaps and will lead to less upward pressure on prices for any *given* output gap. Monetary policy would be more effective (due to more real rigidity). Increases in money supply (decreases in short-term nominal interest rates) would not lead to as much price increases but instead stimulate output more.

Exercise 4. Suppose the bank underestimates potential output, y_n. According to the three equation New Keynesian model, what are the consequences?

Solution 4 (Three Equation NK Model & Deflationary Spirals)**.** If potential output is underestimated, the bank will raise nominal interest rates via the Taylor Rule by more than is required, which will in turn raise the real interest rate by more than is required and so depress output via the IS curve. The depressed output will reduce inflation via the NKPC, which will raise real interest rates further, leading to more depressed output and a deflationary spiral.

Exercise 5. Rewrite the three-equation New Keynesian model, explicitly writing the real interest rate in terms of the nominal rate and expected inflation. If there is an oil shock, and the central bank does *not* follow the *Taylor principle*, explain what happens.

Solution 5 (Three Equation NK Model)**.**

$$\begin{aligned}
\text{[NKIS]}\ y_t &= \frac{\rho}{\theta} - \frac{1}{\theta}(i_t - E_t\pi_{t+1}) + E_t c_{t+1} + g_t + v_t \\
\text{[TR]}\ i_t &= r_n + \pi_t + \gamma(y_t - y_n) + \beta(\pi_t - \bar{\pi}) + \epsilon_t \\
\text{[NKPC]}\ \pi_t &= \phi E_t\pi_{t+1} + \frac{\alpha\delta}{1-\delta}(y_t - y_n) + u_t
\end{aligned}$$

where v_t, ϵ_t and u_t capture movements in y_t, i_t and π_t that are unrelated to what is already included in the equations. If the central bank does not follow the Taylor principle, then $\beta < 0$ and we cannot uniquely determine π_t and i_t (inflation spirals out of control – see figure 1413 in Mankiw). Central banks need to aggressively respond to inflation. With an oil shock, if it is counted as demand shock, then v_t rises pushing up y_t via IS; if the oil shock is counted as supply shock, then u_t rises pushing up π_t via NKPC. Either way, output / inflation rises, then by TR the RHS is higher by $\gamma\frac{\alpha\delta}{1-\delta}$ or β; in the first case, this is because an increase of one for y_t in NKIS would rise π_t by $\frac{\alpha\delta}{1-\delta}$ according to the NKPC. Let's assume it's the second case so NKPC was affected and π_t rises by 1. If $\beta < 0$, then $\frac{di_t}{d\pi_t} = (1+\beta) < 1$ so $r_t - \pi_t$ falls. In turn by NKPC, y_t rises by the absolute magnitude of the fall in r_t times $\frac{1}{\theta}$ so $\frac{\beta}{\theta}$, which raises π_t by $\frac{\alpha\delta}{1-\delta}$ times the rise in y_t, i.e. $\frac{\beta\alpha\delta}{1-\delta}$ according to the NKPC. In turn the rise in y_t and the rise in π_t by NKPC will raise i_t in the TR but less than sufficiently so r_t falls again stimulating further rises in y_t (NKIS) and π_t (NKPC). This becomes an inflationary spiral.

Exercise 6. Consider the three-equation New Keynesian Phillips Curve when there are no shock terms. In the NKPC, assume the coefficient on expected inflation is one. Show that if the bank is a strict inflation targeter (i.e., it only cares about inflation), it also maintains output at potential. (This is called the *divine coincidence*).

Solution 6 (Three Equation NK Model)**.** A strict inflation targeter (only caring about inflation) will set $\gamma \approx 0$ and with $\phi = 1$, we get that

$$\begin{aligned}
\text{[NKIS]}\ y_t &= \frac{\rho}{\theta} - \frac{r_t}{\theta} + E_{c_{t+1}} + g_t \\
\text{[TR]}\ i_t &= r_n + \pi_t + \beta(\pi_t - \bar{\pi}) \\
\text{[NKPC]}\ \pi_t &= E_t\pi_{t+1} + \frac{\alpha\delta}{1-\delta}(y_t - y_n)
\end{aligned}$$

In equilibrium, the central bank sets inflation equal to the target: $\pi_t = \bar{\pi}$, which is constant for all t, so the expected future inflation $E_t\pi_{t+1} = \bar{\pi} = \pi_t$. Using this in the NKPC we get that output will be maintained at potential $y_t = y_n$. With an absence of real rigidities, there is no trade-off in the New Keynesian model between inflation stability and output stability, i.e. we get the 'divine coincidence'. *Aside*: for more, see Blanchard & Gali, 2005 on divine coincidence.

Exercise 7. Suppose the economy is at potential and government expenditure falls. Explain what happens in the basic New Keynesian model presented in class. What happens to the deviation of output from its socially efficient level?

Solution 7 (Government Expenditure in NK Model)**.** Fall in g_t reduces y_t via IS curve, so output falls relative to its socially efficient level.

Exercise 8. The hybrid New Keynesian Phillips curve is given by

$$\pi_t = \gamma(y_t - y_n) + \xi\phi E_t(\pi_{t+1}) + (1-\xi)\phi\pi_{t-1}$$

As ξ tends to one, does it become easier or harder to reduce inflation? What are the implications for inflation dynamics?

Solution 8 (Hybrid NKPC)**.** As ξ tends to one, the NKPC relies less on adaptive (past) inflation and more on rational expectations of future inflation and goes back to the model in class. Relying less on past inflation, it becomes easier to reduce inflation since inflation becomes less persistent. However, now rational expectations of future inflation come into play so there may well be less persistent inflation dynamics but policy must focus on expectations management and open mouth operations in order to influence inflationary expectations.