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## Problem Set 6: Real Business Cycle Model

### Real Business Cycle Theory

**Exercise 1.** Suppose a government decides to raise tax rates permanently. However, it *phases* in the new policy over a number of years. What are the economic effects of this for labour supply?

**Solution 1** (Tax Smoothing). Done in class.

**Exercise 2.** Empirically, *quits* from jobs fall in recessions. Is this observation consistent with RBC theory?

**Solution 2** (Evidence regarding RBC theory). According to RBC-ers, recessions reflect negative fluctuations in technology (the Solow residual), i.e. growth in technology is below its trend. When productivity goes down, it is partially because those who may have quit before and are unproductive do not quit. Layoffs may rise if there is no labour hoarding, or fall/stay constant if there is labour hoarding. You might not want to work more when you are unproductive according to RBC-ers, so the government should not step in to stabilise the economy.

**Exercise 3.** Write down the labour/leisure optimality condition in the RBC model. Over time, wage and consumption grow at approximately the same rate. What does the RBC model predict for the long-run trend in labour hours?

**Solution 3** (Long-run labour trend in RBC). Done in class.

**Exercise 4.** Does RBC theory take the view that recessions are desirable?

**Solution 4** (Desirability of recessions). Done in class.

**Exercise 5.** Suppose the Irish government raises marginal tax rates on wages *temporarily* in the budget. Explain clearly the implications of this policy would within the context of the RBC model.

**Solution 5** (Temporary Marginal Tax Rates). Assume a one period shock to tax rates on wages. If the government were to raise marginal tax rates on wages temporarily, the substitution effect would tend to dominate the income effect and both the intertemporal substitution and capital accumulation channels would propagate the shock downwards negatively; for the capital accumulation channel, the fact that there would be less labour supply means that capital would be less productive (each machine becomes less useful) reducing the demand for capital. Household income falls but temporary nature of the shock means that consumers won't feel that much poorer so will only reduce consumption by a small amount. Since they anticipate that the tax will be temporary, they smoothly adjust their consumption over time so the fall in consumption will be smaller the more temporary the shock is. Labour supply falls initially due to the intertemporal substitution effect but will rise back towards its initial level from the second period onwards due to the substitution effect (when the tax falls back towards its initial value) and the negative income effect. Supply of capital is fixed but as the firms now use less capital and less labour and technology is constant, output will fall temporarily. As people work more from the second period onwards, capital productivity returns to normal and capital demand returns to normal so with constant technology, output returns to normal. To the extent that wages are perfectly flexible, when people initially want to reduce their labour supply as labour taxes rise, there is upward pressure on real wages since labour supply has diminishing marginal productivity. This may induce them not to reduce labour supply too much, but they will reduce labour supply a little since if they did not reduce labour supply then firms would have to pay a higher real wage than initially to compensate workers for the wage tax hike even though there was no increase in productivity (no reduction in labour supply). When capital falls initially, MPK rises due to diminishing marginal productivity of capital. This pushes upwards pressure on rental rates and encourages investment once again.

## SOLUTION

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**Exercise 6.** According to RBC theory, if the government was attempting to stabilize labour input over the business cycle, how should it levy taxes on wage income?

**Solution 6** (Stabilisation Policy and Tax Smoothing). Done in class.

**Exercise 7.** Suppose that, because of budget deficits, the government systematically raises labour taxes in a recession, but lowers them in a boom. What are the implications for labour supply?

**Solution 7** (Procyclical Fiscal Policy). Done in class.

**Exercise 8.** Explain why, the more transitory the shock, the greater the increase in the investment/gdp ratio, when  $A$  rises.

**Solution 8** (Transitory Shocks). Done in class.

**Exercise 9.** Suppose in a two period RBC model, utility takes the form

$$\log c_1 + \log c_2 - \gamma l_1 - \gamma l_2$$

while the lifetime budget constraint is  $c_1 + c_2 = A_1 l_1 + A_2 l_2$ . If  $A_1 > A_2$ , determine  $l_2$ . Use intuition only, and explain answer. Would your answer be the same if utility took the form  $\log c_1 + \log c_2 - \gamma l_1^3 - \gamma l_2^3$ . Explain.

**Solution 9** (Role of Convex Preferences). Done in class.

**Exercise 10.** Suppose the utility function in the RBC model takes the form

$$u(C) = \log C$$

Assume agents may work 12 hours per day. What would labour supply be in equilibrium? Explain how the dynamics of the model would change if there was a temporary, but persistent TFP shock.

**Solution 10** (Role of Labour in Utility Function). Done in class.

**Exercise 11.** Explain what would happen to i) output ii) consumption and iii) labour supply if there was a temporary increase in  $A$  for one period, and the production function was  $Y = AL$ ? Assume output is non-storable.

**Solution 11** (No capital and non-storable output). Done in class.

**Exercise 12.** If shocks to  $A$  were not persistent, would investment still rise if  $A$  rose in a given period?

**Solution 12** (Persistence of TFP Shocks & Investment). Done in class.

**Exercise 13.** What would happen if consumers expected a once-off positive TFP shock *next period*? What implications would there be for the economy *this* period?

**Solution 13** (Once-off TFP Shocks). Done in class.

**Exercise 14.** If there was temporary but persistent increase in  $A$ , what would happen to the rental rate on capital over time.

**Solution 14** (Evolution of Rental Rate of Capital). Done in class.

**Exercise 15.** What happens if there was a *once-off* 1% increase in the capital stock in a given period?

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**Solution 15** (Rise in Capital Stock). An increase in the capital stock in a given period would boost output through the production function but with diminishing returns to capital, the marginal productivity of capital would decline inducing a fall in the rental rate on capital. Since labour demand is given by  $L^d = \left( \frac{(1-\alpha)AK^\alpha}{w} \right)^{\frac{1}{\alpha}}$  and  $K$  increases, labour supply must rise as upwards pressure on real wages would occur, further boosting output through the production function; a rise in capital also boosts the marginal product of labour and thereby raises labour demand and wages in equilibrium. The rise in labour puts upward pressure on the rental rate of capital but the rise in capital would put a downward pressure on the rental rate of capital (reduction in MPK). Output would rise due to  $K$  and  $L$  with  $A$  constant. The temporary nature would spur consumption to rise (SE dominating IE) though this would be small (smoothing / permanent income hypothesis). Since the rise in capital was once-off, labour demand would revert to normal in the next period along with capital (no change in the rental rate of capital). Things revert to normal thereafter.

**Exercise 16.** What happens to labour supply in period 1 if there is a *permanent* rise in productivity?

**Solution 16** (Permanent TFP shocks). Done in class.